Math background self-assessment, CS320

Complete each of the problems below. The spirit of this activity is to make your best attempt to complete each problem using the tools that are at your (mental) fingertips. Googling a specific detail (e.g., "What is Bayes' formula?") is okay, as is using a calculator to work out a specific calculation. However, you should not use people/reference materials/AI to get more substantial help on any of the problems.

If you are completely stuck on a problem, it's okay to simply write *?* or *I don't know* as your answer. After submitting the assignment, you should then take the time to learn or brush up on the material (classmates, the internet and Anna can all help).

Work out your answers by hand first. Then submit your answers to the Moodle quiz (this will allow you to see the correct answers right away rather than waiting for me to grade).

1. Imagine that a patient tests positive for Protein Z. 86% of patients who have Hashing Disease test positive for Protein Z. 10.4% of the overall population tests positive for Protein Z, and only 1 in 10,000 people actually has Hashing Disease. What's the probability that this patient has Hashing Disease? Include 4 significant figures in your answer. (Note: your answer will be between 0 and 1, and significant figures do not include leading zeros. E.g., if my answer was .0020349871, my answer to 4 significant figures would be .002034.)

2. You flip a fair coin and win one point if it's heads and four points if it's tails. What is the expected number of points after one flip?

3. You're playing the same game as in the last question: for every flip of the same fair coin, you win one point if it's heads and four points if it's tails. What is the expected number of points after 50 flips?

4. You are playing a solitaire game in which you are dealt three cards without replacement from a simplified deck of 10 (marked 1 through 10). You win if all your cards are odd or if one of them is a 10.
   (a) How many winning hands are there if different orders are different hands (e.g., drawing 3, 5, 7 is different from drawing 7, 5, 3)?
   (b) What is your probability of winning? (Write your answer to at least 3 decimal places.)

5. Prove by induction that for $n \geq 2$, the sum of integers greater than 0 and less than n is equal to $\frac{n(n-1)}{2}$.

6. Which statements are true for all joint distributions over x and y? (A joint distribution means a probability distribution over two variables.)
   (a) $p(x) = \Sigma_y p(x, y)$
   (b) $p(x, y) = p(x \mid y)p(y)$
   (c) $p(x) = \Sigma_y p(x \mid y)$
   (d) $p(x, y) = p(y \mid x)p(y)$
   (e) $p(x, y) = p(x)p(y)$

7. Which of the following statements are true in general? (i.e., true for all x and y, not just for some special cases)
   (a) $\log(5x) = \log(5) + \log(x)$
   (b) $\log(5^x) = \log(5)\log(x)$
   (c) $2^{x+y} = 2^x 2^y$
   (d) $2^{x+y} = 2^x + 2^y$
   (e) $\log(5^x) = 5x$
   (f) $\log(5^x) = x\log(5)$
   (g) $2^{xy} = 2^x 2^y$

8. A partial derivative with respect to a variable x of an equation that involves multiple variable is the derivative of that equation where x is treated as a variable, and all other variables are treated as constants. For example, consider the following function of three variables:

$$f(x, y, z) = x^2 y + z$$

   The partial derivative with respect to x is denoted as $\frac{\delta f}{\delta x}$:

$$\frac{\delta f}{\delta x} = 2xy$$

   The derivative of $x^2 y$ is just as if we were taking the derivative of x^2 with some numeric value as its coefficient, and the derivative of z is zero, just as if it were a number (since z doesn't involve x).

   Let f be a function of variables x and $w_1, w_2, \ldots, w_n$ and f also involves some constants $b_1, \ldots, b_n$. What is the partial derivative with respect to x of the following function f?

$$f(x, w_1, \ldots, w_n) = \Sigma_{i=1}^{n} w_i (b_i - x)^2$$

9. Sometimes we want to take the derivative of the product of two functions: $f(x) \cdot g(x)$. To do so, we can use the *product rule*. That is, the derivative of $f(x)g(x)$ with respect to x is equal to $f'(x) \cdot g(x) + g'(x) \cdot f(x)$, where $f'(x)$ and $g'(x)$ refer to the derivatives of f and g with respect to x.

   Find the derivative of $x^2 \log x$ with respect to x, where $\log x$ refers to the natural logarithm. Then, using a calculator, evaluate the derivative when $x = 5$. Write your answer to two decimal places.